A simple firing-rate model of working memory and decision-making

Consider two excitatory neural assemblies, that compete with each other through a shared pool of inhibitory neurons. Let \( r_1 \) and \( r_2 \) be their respective population-firing rates, and the total synaptic input current \( I_i \) and the resulting firing rate \( r_i \) of the neural population \( i \) obey the following input-output relationship (\( F - I \) curve):

\[
    r_i = F(I_i) = \frac{aI_i - b}{1 - \exp(-d(aI_i - b))}
\]

which captures the current-frequency function of a leaky integrate-and-fire neuron. The parameter values are \( a = 270 \text{ Hz/nA} \), \( b = 108 \text{ Hz} \), \( d = 0.154 \text{ sec} \).

Assume that the ‘synaptic drive variables’ \( s_1 \) and \( s_2 \) obey

\[
    \frac{ds_1}{dt} = \phi(F(I_1)\gamma(1 - s_1) - s_1/\tau_s) \tag{2}
\]

\[
    \frac{ds_2}{dt} = \phi(F(I_2)\gamma(1 - s_2) - s_2/\tau_s) \tag{3}
\]

where \( \gamma = 0.641 \), and \( \phi = 1 \) except for problem (5). \( I_1 = g_E s_1 - g_I s_2 + I_b^1 + g_{\text{ext}} \mu_1 \), \( I_2 = g_E s_2 - g_I s_1 + I_b^2 + g_{\text{ext}} \mu_2 \). The synaptic time constant \( \tau_s = 100 \text{ ms} \). The synaptic coupling strengths \( g_E = 0.2609 \text{ nA} \), \( g_I = 0.0497 \text{ nA} \) and \( g_{\text{ext}} = 0.00052 \text{ nA} \). Stimulus-selective inputs to populations 1 and 2 are governed by unitless parameters \( \mu_1 \) and \( \mu_2 \), respectively. \( I_b \) is the background input which has a mean \( (I_0) \) and a noise component described by an Ornstein-Uhlenbeck process:

\[
    \frac{\tau_0 dI_{b1}}{dt} = -(I_{b1} - I_0)/\tau_0 + \eta_1(t)\sqrt{\tau_0 \sigma^2} \tag{4}
\]

\[
    \frac{\tau_0 dI_{b2}}{dt} = -(I_{b2} - I_0)/\tau_0 + \eta_2(t)\sqrt{\tau_0 \sigma^2} \tag{5}
\]

where \( I_0 = 0.3255 \text{ nA} \), filter time constant \( \tau_0 = 2 \text{ ms} \), and noise amplitude \( \sigma = 0.02 \text{ nA} \); \( \eta(t) \) is a Gaussian white-noise with zero mean and unit standard deviation.

**Caution:** \( F(I) \) is given in Hz, but in the \( s \)-equations, it should be divided by 1000 so that it has the unit of \( 1/\text{msec} \).
(1) ‘Coin-tossing’ simulations with $\mu_1 = \mu_2 = \mu_0 = 30$ ($c' = 0$). In a decision-making simulation, both $\mu_1$ and $\mu_2$ are presented for a time interval, say from $t_1 = 500$ ms to $t_2 = 1500$ ms ($T = t_2 - t_1 = 1$ sec), and your total simulation time should be much longer (say 3 sec). The decision choice is determined according to which of the two active attractors wins the competition.

Display time courses of $s_1(t)$ and $s_2(t)$, as well as firing rates $r_1(t) = F(I_1)$ and $r_2(t) = F(I_2)$. Also, plot $s_1$ against $s_2$, or $r_1$ against $r_2$ in the ‘phase space’.

In different ($n$) trials (each with a different seed for the random number generator, but always with the same initial condition $s_1 = s_2 = 0.1$), what do you observe? Do you see 50-50 decision outcome if $n$ is large, say $n=100-500$?

(2) Stimulus-specific stimuli are given by $\mu_1$ and $\mu_2$. The ‘coherence level’ is defined as $c' = (\mu_1 - \mu_2)/(\mu_1 + \mu_2)$. For example, if $\mu_1 = 0.84$ and $\mu_2 = 0.8$, then $c' = 0.0244$ or 2.44%. Repeat (2) with several $c' = 0.032, 0.064, 0.128, 0.256, 0.512, 0.85, 1.0$ (for example, with $\mu_1 = \mu_0(1 + c')$ and $\mu_2 = \mu_0(1 - c')$). Plot the ‘psychometric function’, namely the percentage of correct decisions (choice=1 is correct if $\mu_1 > \mu_2$) as a function of log($c'$).

(3) Reaction time task. Set a firing threshold (e.g. $\theta = 15$ Hz, but adjust it if necessary). In any trial, the decision is made whenever one of the two neural populations reaches this threshold first. Run simulations over many trials for each $c'$ as in (2).

(a) Show sample time courses of firing rates for different coherence levels.

(b) Plot the psychometric function, namely the trial-averaged reaction time as a function of log($c'$).

References